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A Model of the Beam-Pellet Interaction

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1 Introduction

In this note we describe a detailed model for the interaction of a circulating proton or anti-proton beam with a frozen hydrogen pellet. Such a pellet target station [1, 2, 3] is integrated in the WASA [4] detector at the CELSIUS storage ring [5] at the The Svedberg Laboratory in Uppsala, Sweden. The target is in operation since 1999. A similar pellet target is proposed for the PANDA detector that will find its home in the HESR storage ring [6] at GSI, Germany. In CELSIUS the typical storage time is on the order of a few minutes with beam lifetimes of the same order of magnitude despite the lack of cooling, but with RF turned on, to compensate for the energy lost in the target. The demands on lifetime in HESR are drastically higher, because the circulating beam is made of anti-protons which are very expensive to produce. Only about 10^7 anti-protons per second can be produced on average in the proposed GSI accelerator complex. This implies that the anti-proton loss rate in the HESR ring has to be lower than the production rate.

The interaction of the protons with the pellets is characterized by rare hits of the pellets, which themselves are moderately thick. The pellet diameter is typically 20 to 30 μm. This must be compared to, for example, gas scattering events, where the circulating beam interacts with individual rest-gas atoms, but not with small solids, as is the case for the interaction with the pellet. On the other hand the pellets are not yet really thick solids that allow using, for example, Landau's theory of energy loss straggling. The beam-pellet interaction is thus in-between several well-established theories, that are commonly used in accelerator physics [7, 8]. We will try to solve this dilemma by using algorithms commonly found in detector simulation tools, such as GEANT [9] and present alternate algorithms that re-implement the methods found in GEANT. These algorithms can be integrated in other codes to later simulate the time behavior and the equilibrium of the beam, both as a collection of non-interacting single particles or as a collective ensemble.

We now delve into the description of the pellets and how they affect the beam and describe the algorithms as we go along.

2 What is the target?

The pellets are small spheres of frozen hydrogen with a typical diameter d of about $d = 2R = 30\,\mu$m, although the size can vary by a factor 2 or so. The density of frozen or solid hydrogen is taken from [10] to be $\rho_H = 0.0708\,\mathrm{g/cm^3}$ and the radiation length is 8.9 m. Using these number the mass of one pellet amounts to $m = 4\pi R^3\rho_H/3 = 10^{-9}$ g. The mass of a single proton is $1.67\,10^{-24}$ g such that there are about $6\,10^{14}$ hydrogen atoms in a single pellet.

These pellets are vertically shot through the beam at a rate of about 60 kHz such that the time between pellets is $16\,\mu$s. The speed of a pellet is on the order of 60 m/s such that one pellet follows the other after about 1 mm. The pellets do have a small transverse spread and cover about ± 2 mm horizontally. These parameters are only approximate as most of them depend on detailed tuning of the source and in particular the used vacuum injection nozzle.

We need to note that the effective thickness t_e of the spherical pellets is given by the volume of a sphere of radius R which is $4\pi R^3/3$ divided by its projected cross-section which is πR^2 such that we obtain for the effective thickness $t_e = 4R/3$ or $t_e = 2d/3$.

We will briefly report the radiation length of liquid hydrogen in various units, all are related to the density $\rho_H = 0.0708\,\mathrm{g/cm^3}$

- $X_0 = 890$ cm.
- $X_0 = 63.05\,\mathrm{g/cm^2}$.
- $X_0 = 3.77 \times 10^{25}\,\mathrm{atoms/cm^2}$.

The values are taken from ref. [10]. These radiation lengths will facilitate calculation of, for example, the rms scattering angle.

In a simple approach to describe the fate of the colliding proton we can use the effective thickness t_e. This approach can, however, be refined by subdividing the pellet into, for example, ten concentric parts and tabulating all relevant parameters, that we will describe below, for the subdivisions. Since the outward concentric slices are larger they have a higher probability of being hit. The central slice, where the pellet is thickest, has the lowest probability. We model this by

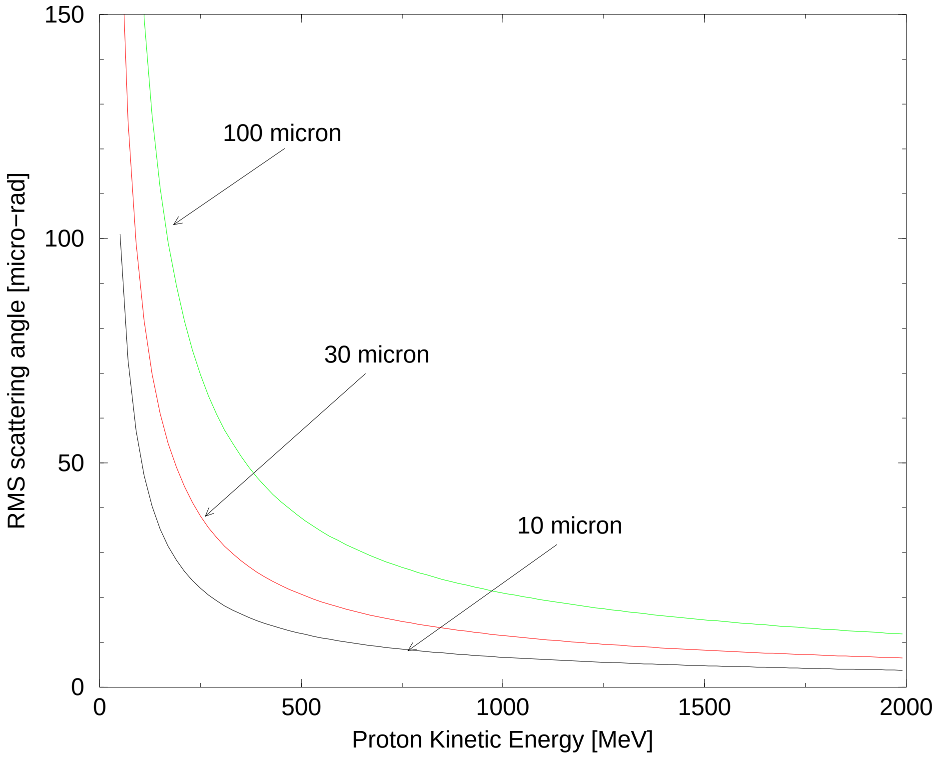

Figure 1: *Transverse scattering angle as a function of beam energy for pellet diameters of 10, 30, and* $100\,\mu m$. *This case is relevant for CELSIUS.*

calculating a random number between 1 and 100. If the random number is 1, we choose the tabulated parameters for the thickest slice, if it is between 2 and 4 the next set of parameters, between 5 and 9 the next, and so forth. In this way we assign the proper weights to the probabilities of hitting the pellet where it has a particular thickness. The algorithm is rather fast, because the table can be calculated before-hand and choosing the slice involves sampling a random number and a table lookup.

3 What does the pellet do to a proton?

When a proton or another particle hits a pellet it experiences both transverse and longitudinal kicks, i.e. it is scattered transversely by an angle Θ and looses energy such that the momentum of the incident particle is changed. Both effects are carefully simulated in detector simulation codes such as GEANT [9].

Table 1: Pellet Properties for 1360 MeV protons ($\beta = 0.912,\ \gamma = 2.45$).

Property	Symbol	10 μm	30 μm	100 μm	Units	Comments
target						
eff. thickness	t	6.7	20	67	μm	2/3 diameter
ionization energy	I	16	16	16	eV	
transverse						
rms angle	θ_0	5.1	8.9	16.2	μrad	Molière
	χ_{cc}	0.149	0.149	0.149	MeV/$\sqrt{cm}$	Molière
	χ_c	2.00	3.48	6.35	10^{-6}	Molière
	χ_α	2.13	2.13	2.13	μrad	Molière
	b_n	949	949	949	1/cm	Molière
# scatters	Ω_0	0.76	2.28	7.59		Molière
longitudinal						
energy loss	ΔE	206	618	2060	eV	
	ξ	9	26	89	eV	Landau
rel. mom. loss	$\Delta p/p$	108	320	1075	10^{-9}	
rel. straggeling	$\delta p/p$	5	14	45	10^{-9}	
	E_{max}	5.1	5.1	5.1	MeV	
	κ	1.7	5.1	17.0	10^{-6}	Landau
# excitations	$< n_1 >$	7.7	23.2	77.2		Urban (r=0.4)
excitation energy	E_1	16	16	16	eV	Urban (r=0.4)
# excitations	$< n_2 >$	0	0	0		Urban (r=0.4)
excitation energy	E_2	10	10	10	eV	Urban (r=0.4)
# ionizations	$< n_3 >$	0.4	1.2	4.1		Urban (r=0.4)
mean ionization energy	E_3	202.7	202.7	202.7	eV	Urban (r=0.4)
# excitations	$< n_1 >$	0	0	0		Urban (r=1.0)
excitation energy	E_1	16	16	16	eV	Urban (r=1.0)
# excitations	$< n_2 >$	0	0	0		Urban (r=1.0)
excitation energy	E_2	10	10	10	eV	Urban (r=1.0)
# ionizations	$< n_3 >$	1.02	3.05	10.16		Urban (r=1.0)
mean ionization energy	E_3	202.7	202.7	202.7	eV	Urban (r=1.0)
hit probability	P_{hit}	$1.8\,10^{-5}$	$1.8\,10^{-4}$	$1.8\,10^{-3}$		
turns between hits		56000	5600	560		

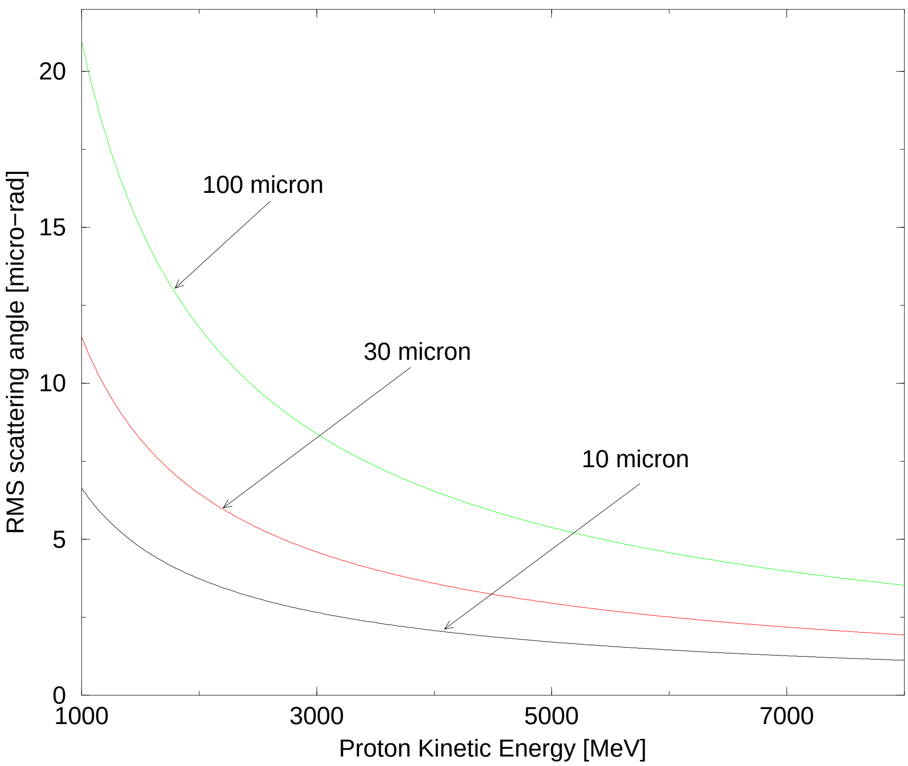

Figure 2: *Transverse scattering angle as a function of beam energy for pellet diameters of 10, 30, and* $100\,\mu m$*. This case is relevant for HESR.*

3.1 Transverse

We will consider the transverse effect first and follow the discussion in the GEANT manual [9]. The particle is scattered transversely due to multiple small angle scattering off the Coulomb field of the targets nuclei. The theory to describe these processes is called Molière scattering [10]. The scattering is not Gaussian, with tails more heavily populated than derived from a Gaussian distribution. In GEANT, however, a equivalent Gaussian model, that can be used for rough estimates is described. The in-plane rms scattering angle θ_0 given by

$$\theta_0 = 2.557\chi_{cc}\frac{\sqrt{t}}{\gamma\beta^2 mc^2} \ . \tag{1}$$

The parameters β and γ are are the normalized speed and energy of the incident beam. The Molière parameter χ_{cc} is given by

$$\chi_{cc} = 0.39612\sqrt{\frac{Z_m(Z_m+1)}{A_m}}\sqrt{\rho_H} \tag{2}$$

where the subscript m denotes the material properties. For hydrogen we have $Z_m = A_m = 1$ such that $\chi_{cc} = 0.56\sqrt{\rho_H}$ where ρ_H is the density of liquid hydrogen

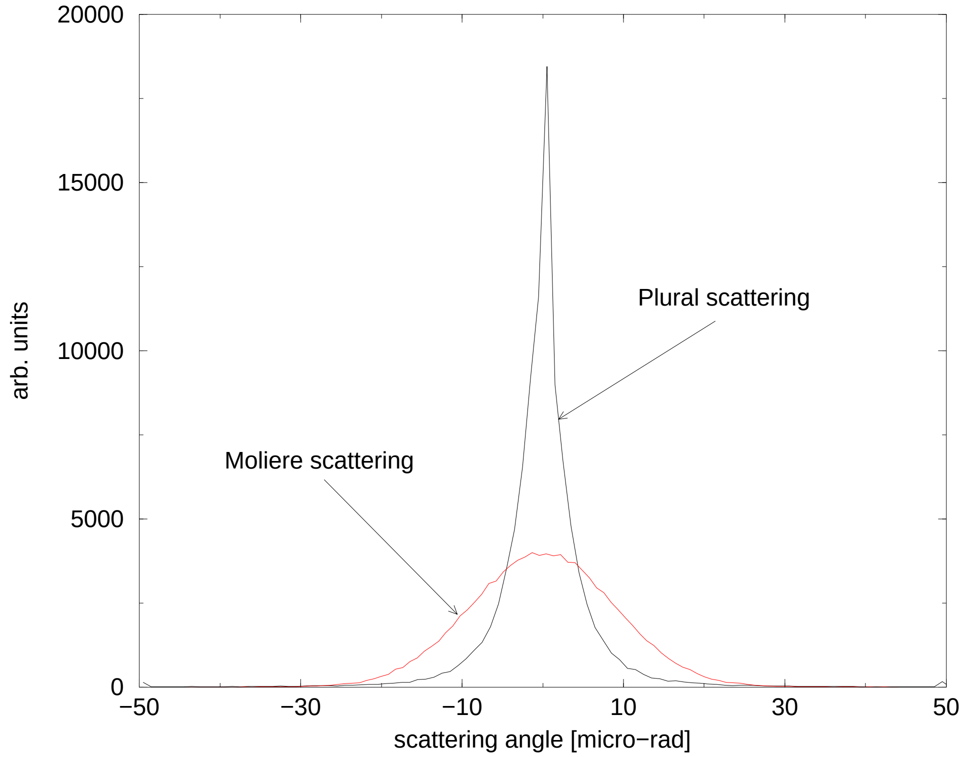

Figure 3: *Molière versus plural scattering for 1360 MeV protons scattered off pellets with* $30\,\mu$m *diameter.*

in g/cm^3. The rms scattering angle as a function of the beam energy for pellet diameters of 10, 30, and 100 μm are shown in Fig. 1 for energies relevant for CELSIUS [5] and in Fig. 2 for energies relevant for HESR [6]. Note that we have collected all relevant parameters for pellet diameters of 10, 30, and 100 μm in table 1.

The argumentation in the previous section is not quite accurate because the pellet target is moderately thick and the number of scatterings Ω_0 within a given pellet is around unity as can be calculated from the formulae in the GEANT manual. We have

$$\Omega_0 = b_c Z_{inc}^2 \frac{t}{\beta^2} \tag{3}$$

with b_c given by

$$b_c = 6702.33 \rho_H Z'_s e^{(Z'_E - Z'_x)/Z'_s} \; . \tag{4}$$

For hydrogen we have $Z'_s = Z(Z+1)/A = 2$ and $Z'_E = 0$ as well as $Z'_x \approx 0$ such that we get $b_c = 13400\rho_H \approx 949/\text{cm}$. Inserting in eq. 3 we get $\Omega_0 \approx 2$ which is outside the range of validity of the Molière theory. The GEANT manual states that we have to use the theory for "Plural Scattering" described in section PHYS328. To utilize that we first have to calculate the average number of scatters

$\hat{n}$ inside the medium which is sampled from a Poisson distribution with mean $1.167\Omega_0$ and then obtain the individual scattering angles by sampling them from the following distribution which is basically the rescaled Rutherford cross-section for a screened Coulomb potential with screening angle χ_α

$$\frac{dN}{d\theta} = \chi_\alpha^2 \frac{2\theta}{(\theta^2 + \chi_\alpha^2)^2} \ . \tag{5}$$

One can generate random number sampled from the distribution shown in eq. 5 by the following recipe

$$\hat{\theta} = \chi_\alpha \sqrt{\frac{1}{\eta} - 1} \tag{6}$$

where η is a random number uniformly distributed in the interval between 0 and 1. The screening angle χ_α is given by

$$\chi_\alpha^2 = \frac{\chi_c^2}{1.167\Omega_0} = \frac{\chi_{cc}^2}{1.167 b_c p^2 c^2} = \frac{2.25\,10^{-11}}{\beta^2\gamma^2} \tag{7}$$

Note that θ is the azimuthal angle (comparable to the latitude on the globe), the other angle ϕ (comparable to the longitude) is unknown or arbitrary. We need the projection of the scattering angle into the horizontal (or vertical) plane. We accomplish this by choosing a random phase angle $\hat{\phi}$ between 0 and 2π and multiply the angle $\hat{\theta}$ by $\sin\hat{\phi}$ to obtain the projection into the horizontal direction and by $\cos\hat{\phi}$ for the vertical direction.

In the algorithm we thus first have to calculate the number of scatters from the Poisson distribution which will be $\hat{n}$ and then run $2\hat{n}$ times a uniform random number generator to get a group of η and random angles $\hat{\phi}$. Adding up the corresponding $\hat{\theta}$ will yield the realization of the scattering angle for this encounter of a proton with a pellet.

In order to compare the more accurate plural scattering theory with the – in this context "wrong" – Molière theory we run a simulation where we average the encounter of 100 000 protons with energy 1360 MeV with 30 μm pellets and show the resulting distributions. For the Molière simulation we calculate θ_0 according to eq. 1 and use a Gaussian random number generator. In this case we have $\theta_0 = 8.9\,\mu$rad. The curve labeled Moliére scattering in Fig. 3 shows a histogram of the resulting scattering angles which obviously is Gaussian. For the "Plural Scattering" simulation we implement the algorithm described in the previous paragraph. We clearly observe that the resulting histogram is much more narrower and the peak is more pronounced which is due to the small number of scatters Ω_0 such that when sampling the random number of scatters $\hat{n}$ one frequently obtains zero as the number of scatters in this particular realization. This implies that the proton traverses the pellet without transverse scattering. On the other hand, if $\hat{n}$ is different from zero, the scattering angle $\hat{\theta}$ is sampled from the distribution given in eq. 5 which has pronounced tails that decay as $1/\theta^3$ which is larger than a Gaussian for sufficiently large scattering angles θ. This implies that large scattering angles are more likely to occur in the realistic case. The cause for these large scattering angles are, of course, small impact parameters or a close fly-by of the beam-proton at a pellet-proton.

We note in passing that the probability distribution function given in eq. 5 does not have a true rms-width, because calculating the rms will lead to a logarithmic divergence.

3.2 Longitudinal

The protons in the beam loose a moderate amount of energy due to ionization and excitation of hydrogen atoms in the pellet. The average energy loss can be calculated from the Bethe-Bloch formula. The energy loss is a stochastic process and only the average loss is described by the Bethe-Bloch formula. The energy lost by particles will be distributed around the average loss. This spreading is called *energy straggling.* For targets of moderate thickness there are theories that describe the energy distribution. There is a rms model for straggling which is limited to rather thick targets or, equivalently passing through a thin target a very large number of times. Other theories are due to Landau and Vavilov [9]. We will discuss the limited validity of those theories and then present an alternative approach in which the ionization and excitation process is simulated with a direct Monte-Carlo method due to Urbán [9].

The regimes of validity of the various theories can be described by the mean energy transfer ξ, featured in the Landau theory and given by

$$\xi = C\frac{\rho\delta x}{\beta^2} \qquad \text{with} \qquad C = 153.4\,\mathrm{keV}/(\mathrm{g/cm}^2) \tag{8}$$

for protons and hydrogen pellets. The other relevant parameter is the ionization potential $I \approx 16Z^{0.9}$ and the maximum kinematically accessible energy transfer to the electrons $E_{max} = 2m_e\beta^2\gamma^2/(1+2\gamma m_e/m_p+(m_e/m_p)^2) \approx 1\,\mathrm{MeV}$. The central parameter is the ratio $\kappa = \xi/E_{max}$ of mean energy ξ and E_{max}. A large value of κ, the typically quoted value is $\kappa > 10$, implies that the material is rather thick and consists of many individual collisions, because each energy transfer is considerably less than E_{max}. Many of those collisions have to contribute in order to account for the mean energy loss ξ. The sum of many small random fluctuations will always produce a Gaussian random distribution as the central limit theorem implies with the rms straggling width σ_E is given by $\sigma_E^2 = \xi E_{max}(1-\beta^2/2)$. The limit $\kappa > 10$ thus implies a lower limit for $\rho\Delta x$ for the target thickness, or conversely, the number of turns that the target must be traversed. For $\kappa = 10$ we have $153.4\rho\Delta x = 10000\,\mathrm{keV}$ when taking $\beta \approx 1$. Solving for the integrated target density we obtain $\rho\Delta x \approx 65\,\mathrm{g/cm}^2$ or, when dividing by the proton mass, $\rho\Delta x \approx 4\times10^{25}$ atoms/cm^2. Assuming a target density of 10^{15} atoms/cm^2 we have to look at time scales of 4×10^{10} turns or about 15 000 seconds or 4 hours if we want

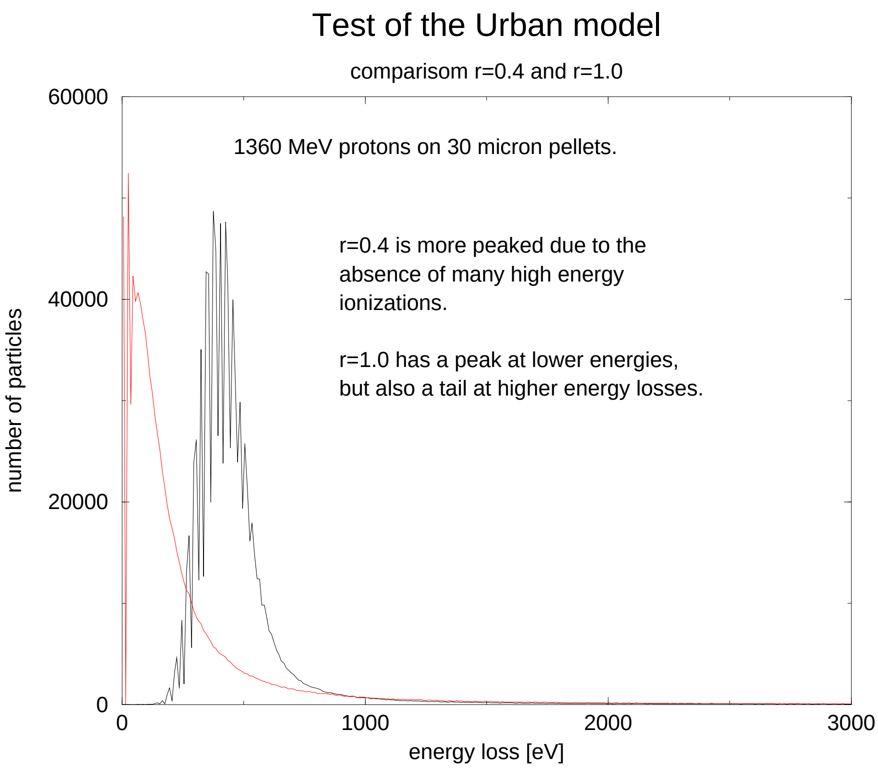

Figure 4: *Energy loss distribution for 1360 MeV protons impacting on* $30\,\mu m$ *pellets simulated by the Urbán model with parameter* $r = 0.4$ *and* $r = 1.0$. *The curves are based on one million proton hits.*

to apply the Gaussian model to describe the beam-target interaction in CELSIUS. On shorter time scales we need to take the detailed energy loss distribution into account. Note that this argumentation tacitly assumes the absence of other forces that act on shorter time scales such as electron cooling or intra-beam scattering.

The range of $0.01 < \kappa < 10$ which corresponds to time scales of a few seconds to a few hours in CELSIUS can be described by the Vavilov theory [11] and shorter time scales which correspond to $\kappa < 0.01$ are the domain of the Landau theory [12]. In his derivation Landau made the assumption that the number of collisions inside the target material is high which translates that the mean energy loss ξ must be much larger than the ionization energy I. For $30\,\mu$m pellets we can use the values from table 1 and find $\xi/I \approx 2$ which is clearly not large. Despite these shortcomings and the fact that these theories are not applicable to short time scales we have tabulated the relevant parameters in table 1.

We have just shown that the average number of ionizations per pellet traversal is of order unity which implies that the assumptions implicit in the Landau theory are not valid. Moreover, the energy loss of charged particles is not only due to ionization processes, but also due to excitations of the atomic levels within the

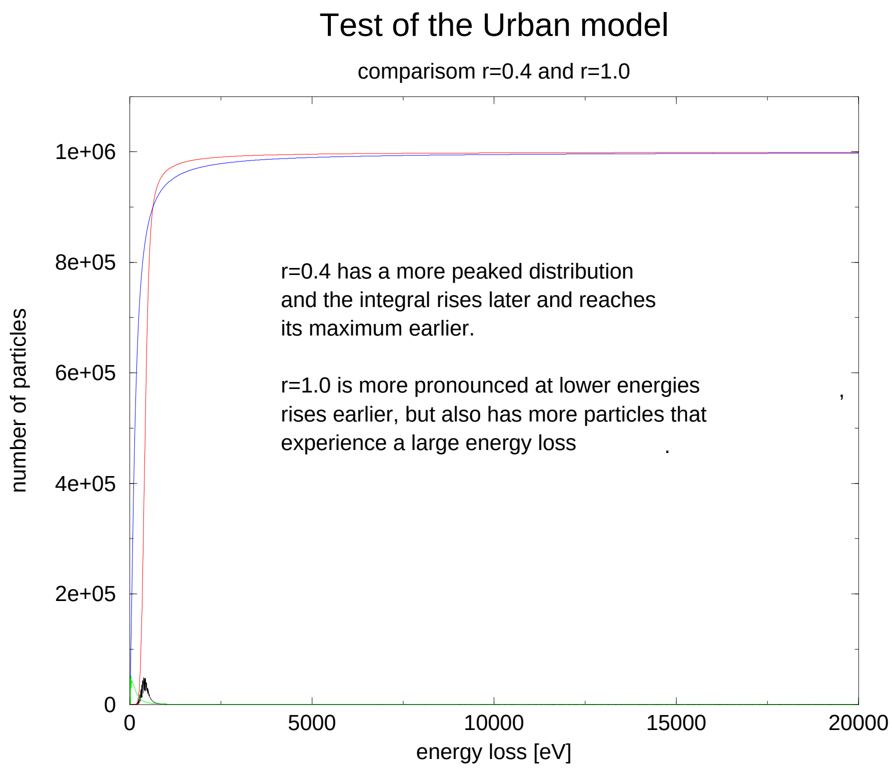

Figure 5: *The integral of the curves in Fig. 4 shows the number of particles which encountered energy loss less than the value on the abscissa.*

hydrogen atoms. In a moderately thick target this contribution can be significant as is claimed in the GEANT manual where an alternative method, developed by *Urbán* is suggested. In this model the ionization and excitation process is directly simulated by a Monte-Carlo process. The ratio between the energy loss due to ionization versus that by excitation can be adjusted by a free parameter r. In GEANT this parameter is set to $r = 0.4$ which means that only 40 % of the energy loss is accounted by ionizations and 60 % by excitations in a simple two level system (for hydrogen, only one level is used, though). Since the excitation energy is only on the order of a few eV while the average ionization energy is on the order of a few 100 eV the distribution for $r = 0.4$ is more centered with smaller tails compared to the distribution where all energy loss is accounted by ionization ($r = 1.0$). Fig. 4 shows a simulation where a million protons with kinetic energy of 1360 MeV are scattered in a pellet of 30 μm diameter. The average energy loss is 618 eV as can be seen from table 1. The two distribution differ markedly. In the case $r = 0.4$ it is peaked slightly below 500 eV, has a moderate tail towards higher losses. The curve for $r = 1.0$ is peaked towards lower energies and has a more pronounced tail. We interpret this by the fact that the average energy loss is made up solely of ionizations which have a much higher energy loss. Thus the

$r = 1.0$-curve consists of many large random numbers, whereas the $r = 0.4$-curve consists of some large random numbers plus a large number of small random numbers which have a smoothing effect on the curve.

In Fig. 5 we display the integral of the curves in Fig. 4. The two curves show the number of particles that experience a loss smaller than the energy loss indicated on the abscissa. The $r = 1.0$-curve rises first and then approaches the maximum somewhat slower than the $r = 0.4$-curve. The tails at higher energy losses towards the right on Fig. 5 agree fairly well, though.

We will now briefly recapitulate the description of the Urbán algorithm as explained in the GEANT manual which is modelled by a simple atomic system with two excitation levels and by ionization. As discussed above the parameter r selects the fraction of the total average energy loss that is attributed to ionization. Ionization losses are characterized by a $1/E^2$ distribution up to the kinematically accessible electron energy E_{max}. that is shown in table 1. The minimum ionization loss that the proton can experience is the the ionization energy $I \approx 16\, Z^{0.9}$ and the maximum is $E_{max} + I$. The $1/E^2$ distribution normalized to unity in this interval is

$$g(E) = \frac{I(E_{max} + I)}{E_{max}} \frac{1}{E^2} \qquad \text{with} \qquad \int_I^{E_{max}+I} g(E) = 1 \ . \tag{9}$$

The average energy loss per ionization process $\bar{E}_{ion}$ is thus

$$\bar{E}_{ion} = \int_I^{E_{max}+I} g(E) E dE = \frac{(E_{max} + I)I}{E_{max}} \ln\left(\frac{E_{max} + I}{I}\right) \tag{10}$$

In order to generate random numbers following the distribution $g(E)$ with the mean given by eq. 10 we have to build a random number generator that transforms uniformly distributed random numbers $0 < \xi < 1$ to those with the distribution $g(E)$. This is accomplished by inverting the cumulative distribution function $\int_I^E g(x)dx$ and arrive at

$$E_r = \frac{I}{1 - \xi E_{max}/(E_{max} + I)} \ . \tag{11}$$

where E_r is a random number that, when binned properly, will produce the distribution $g(E)$. For a full simulation we now need to know how many such ionizations per pellet traversal will occur.

When distributing the fraction r of the total ionization losses which are given by the Bethe-Bloch loss dE/dx of the ionizations in a layer of thickness t we estimate the average number of ionizations $\bar{n}_i$ by dividing the total losses $r(dE/dx)t$ by the average loss per ionization given by eq. 10 and arrive at

$$\bar{n}_i = r \frac{E_{max}(dE/dx)t}{(E_{max} + I)I \ln[(E_{max} + I)/I]} \ . \tag{12}$$

In order to generate random numbers that represent the ionization loss we first have to produce a random number n_i sampled from a Poisson distribution with average $\bar{n}_i$ and then produce n_i random numbers for the individual ionization processes. This can be represented as

$$E_{ion} = \sum_{k=1}^{n_i} \frac{I}{1 - \xi_k E_{max}/(E_{max} + I)} \ . \qquad (13)$$

Note that the thickness of the pellet layer enters through the number of ionizations n_i which itself is a random number, but is produced in such a way as to reproduce the desired fraction r of the Bethe-Bloch losses.

Now we need to turn to the excitation losses which account for the $1 - r$ fraction of the total losses. In the general case the Urbán model uses a two-level system. For systems with $Z \leq 2$ it uses, however, a single level system that has oscillator strength $f = 1$ and energy E_1 equal to the ionization energy I. The average number of excitations $\bar{n}_1$ of this level is then given by the fraction $1 - r$ of the Bethe-Bloch losses divided by E_1. In the simulation we sample a random number n_1 from a Poisson distribution with mean $\bar{n}_1$ and multiply that by E_1 and use that to describe the excitation losses E_e during one pellet traversal.

The total losses ΔE that a beam particle experiences are given by the sum of the excitation and ionization losses as given in eq. 13 and read

$$\Delta E = n_1 E_1 + \sum_{k=1}^{n_i} \frac{I}{1 - \xi_k E_{max}/(E_{max} + I)} \ . \qquad (14)$$

with n_1 and n_i sampled from Poisson distributions with mean $(1-r)(dE/dx)t/E_1$ and $\bar{n}_i$ from eq. 12, respectively. The random variables ξ_k are uniformly distributed in the interval between zero and unity.

4 Conclusions

We discussed a method to realistically model the beam-pellet interaction by describing an algorithm for a “random number generator” that produces the transverse scattering angles and the energy losses appropriate for a moderately thick pellet target. Furthermore, we discussed the limitations of the conventionally applied methods that use rms scattering, both in the transverse and the longitudinal dynamics.

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