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Analytical Design of a Confocal Resonator

T. Lofnes, V. Ziemann (Ed.), The Svedberg Laboratory, Uppsala, Sweden

A. Ferrari, Department of Radiation Sciences, Uppsala University, Sweden

ABSTRACT

A confocal resonator may be used as a pick-up for frequencies in the multi-GHz region, in order to monitor the bunch spacing and/or the bunch length in the CTF3 drive beam. In this note, we collect some formulae regarding the design of a confocal resonator in order to facilitate the estimation of relevant parameters in a later more careful numerical study.

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1 Introduction

During the operation of the CLIC Test Facility CTF3 Preliminary Phase, a new method was tested to monitor the bunch frequency multiplication in the combiner ring. A coaxial pick-up and its read-out electronics were designed and mounted in the CTF3 ring in 2002 to allow comparison of the amplitudes of five harmonics of the fundamental beam frequency (3 GHz) while combining bunch trains. The commissioning of this monitor was a successful proof of principle for this new method [1, 2]. However, two limitations were identified. The first one was that the rise time of the read-out electronics was longer than the time extension of the bunch trains (about 6.6 ns). In the next stages of CTF3, the bunch trains will be 140 ns long and this limitation will thus disappear. The second limitation was the presence of waveguide modes, which were excited by discontinuities upstream in the beam pipe and which were propagating in the wake of the electron bunches, leading to a distortion of the signal coming out from the monitor. The electromagnetic field that the bunches carry with them is a quasi-TEM mode, while the other travelling waveguide modes are TE or TM fields. It was suggested earlier that a confocal resonator like pick-up could discriminate between the quasi-TEM mode of the beam and the parasitic TE and TM fields [3].

In this note, we make analytical investigations of the geometry, the electromagnetic fields and the Q -value of such a device, by using equations that we found in the literature, in order to get some order of magnitude estimates, which will then allow to guide us in later numerical studies. Here, we focus on a confocal resonator for microwaves with a wavelength of 20 mm, which corresponds to a frequency of 15 GHz. We believe that such a device should be easy to manufacture and test. However, this restriction is not crucial, because all quantities with the dimension of a length directly scale with the wavelength. The corresponding linear dimensions for a 100 GHz resonator would simply be smaller by a factor 100/15. Other quantities that such as Q -values may change, however, by a different amount.

In section 2, we give a review of the equations describing the electromagnetic fields in a confocal resonator, with emphasis on the fundamental mode. Then, in section 3, we compute the various losses of such a cavity. Section 4 aims at optimizing the geometry of the confocal resonator at 15 GHz. In sections 5 and 6, we discuss the coupling of the resonator to the quasi-TEM field of the bunched beam and to the waveguide modes propagating in the beam pipe, respectively. Finally, conclusions are drawn in section 7.

2 Confocal Resonator

The fields in a confocal optical resonator were originally investigated in the early 1960s when the first laser oscillators in the microwave and optical regimes appeared. One of the first papers was written by Fox and Li [4] and, later, various authors developed the theory further. A very readable tutorial is given in [5] and an overview of the formulae can be found in [6], which we will closely follow in this report.

The geometry of a confocal resonator is shown in Fig. 1, where the quantities used in the following of our study are also defined. In cylindrical coordinates the paraxial solution of the wave equation that describes waves travelling between the round mirrors is described by gaussian beams modulated by associated Laguerre polynomials, see eq. 1.

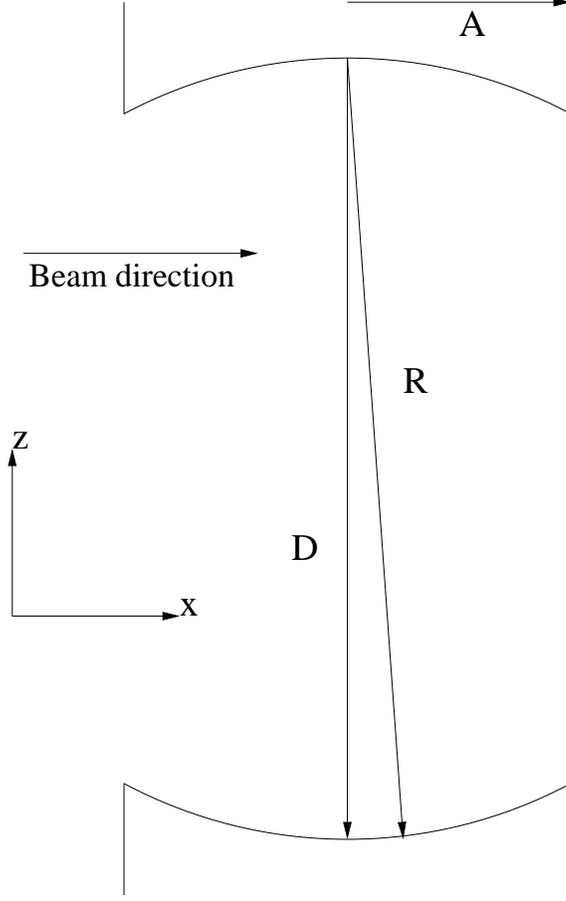


Figure 1: Geometry of a confocal resonator with a definition of the used quantities.

$$\begin{aligned}
u(r, \phi, z) &= A(r, \phi, z) \exp \left[-i \left(kz + \frac{kr^2}{2R(z)} - \Theta(z) \right) \right] \\
A(r, \phi, z) &= \frac{w_0}{w(z)} \left(\frac{r}{w(z)} \right)^m L_n^m \left(\frac{2r^2}{w^2(z)} \right) \exp \left(-\frac{r^2}{w^2(z)} \right) \cos(m\phi) \\
w^2(z) &= w_0^2 \left(1 + \left(\frac{z}{z_0} \right)^2 \right) \\
R(z) &= z \left(1 + \left(\frac{z_0}{z} \right)^2 \right) \\
\Theta(z) &= (1 + 2n + m) \arctan \left(\frac{z}{z_0} \right) \\
z_0 &= \frac{kw_0^2}{2} = \text{Rayleigh length} \\
k &= \frac{2\pi}{\lambda}
\end{aligned} \tag{1}$$

where w_0 is the waist radius of the optical mode, L_n^m is the associated Laguerre polynomial, and λ is the wavelength. Note that the fundamental mode with $n = m = 0$ is gaussian transversely with a rms width $w(z)/\sqrt{2}$. Here, we choose the coordinate system such that the waist is at $z = 0$.

In our design we choose the resonator to be confocal, which means that the distance D between the mirrors is equal to their radius of curvature R , which is also approximately the radius of curvature of the wave fronts. In order to have vanishing fields on the mirrors we require that the cosine of the phase factor vanishes. For the fundamental $n = m = 0$ mode and $r = 0$ this condition leads to

$$\left(l + \frac{1}{2}\right)\pi = kz - \Theta(z) = kz - \arctan\left(\frac{z}{z_0}\right) \quad \text{at } z = D/2 \quad (2)$$

where l is an integer. Solving for the mirror distance D at $r = 0$ we obtain

$$D = \left(l + \frac{3}{4}\right)\lambda. \quad (3)$$

If we choose for example $l = 3$ we obtain a mirror distance of 75 mm, for a wavelength of 20 mm. The choice of this value is motivated more clearly further in this paper.

Having found the distance D between the mirrors and also the radius $R = D$, we can calculate the minimum waist w_0 from

$$w_0^2 = \frac{D\lambda}{2\pi}. \quad (4)$$

For our configuration, w_0 is 15.45 mm, which is of the same order as the wavelength λ .

Having estimated all parameters needed in order to calculate the electric field and the intensities in various planes, we start by plotting the electric field along the z -axis in Fig. 2. We observe that the electric field vanishes at the ends i.e. $z = \pm 37.5$ mm, where the mirrors are located, as expected.

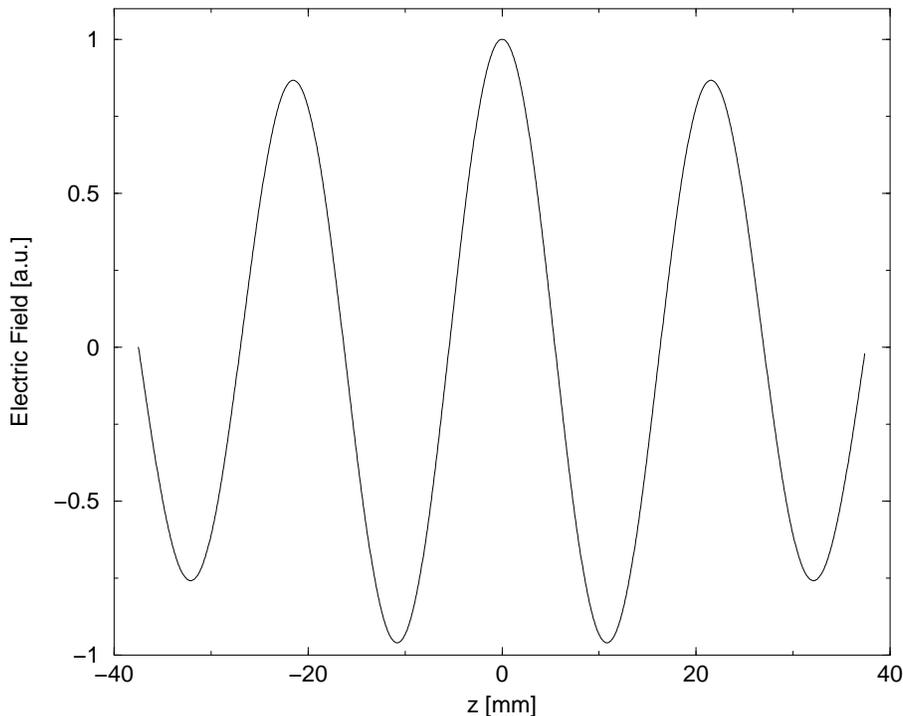


Figure 2: *Electric field on axis, where the mirrors are on the left-hand side and on the right-hand side.*

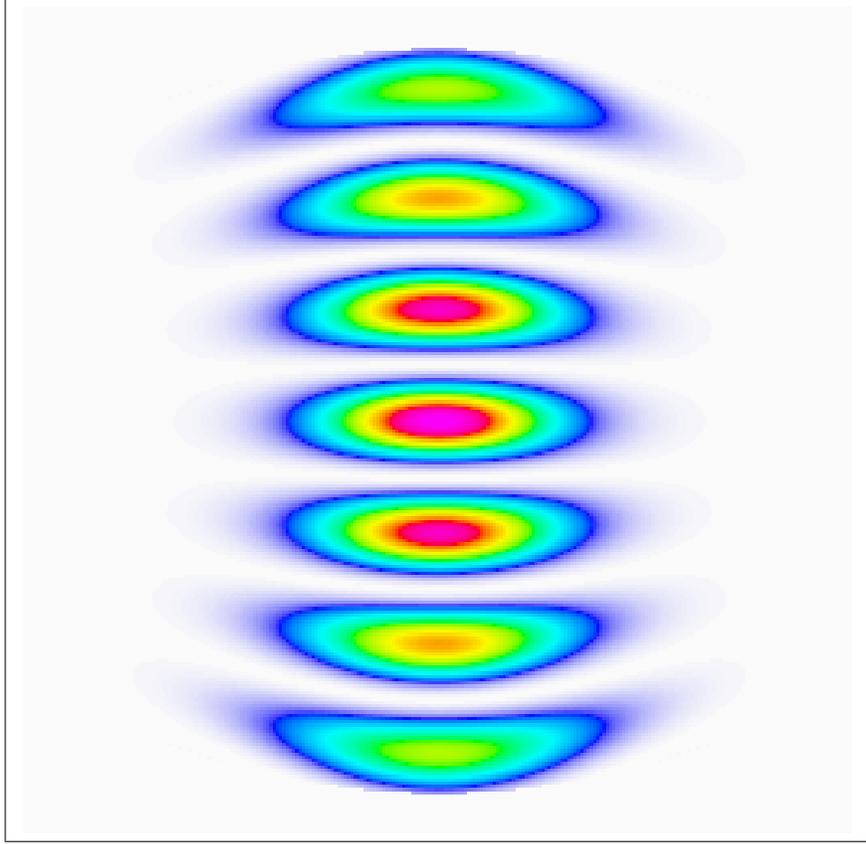


Figure 3: *Intensity of the $(0,0)$ mode in the $x-z$ plane, horizontal= x , vertical= z . The area covered is ± 40 mm in either direction. The mirrors are at the top and at the bottom.*

In Fig. 3 we show the intensity distribution (i.e. the squared electric field) in the $x - z$ plane. The mirrors are at the top and at the bottom, and the z -axis goes from top to bottom. The horizontal direction corresponds to the x -coordinate, which coincides with the variable r in eq. 1. The intensity maxima are clearly visible and there is one on axis. The geometry of the confocal resonator with its two half-domes perpendicular to the direction of the beam together with the modal pattern shown in Fig. 3 makes it clear that the resonator modes are a sort of trapped modes, which are excited by the passing beam.

In Fig. 4 we display the intensity distribution in the $x - y$ plane at $z = 0$ for various modes (n, m) . In the top left plot, we show the $(0, 0)$ plain gaussian mode. The top right plot displays the $(0, 1)$ mode, which has a two-fold azimuthal symmetry. The left graph in the middle row shows a $(1, 0)$ mode and has an extra zero between the maximum in the center and the circular ring. The right graph in the middle row shows a $(1, 1)$ mode with a single radial zero and a two-fold azimuthal symmetry. The bottom row shows the $(2, 0)$ and $(2, 1)$ mode with two radial zeros and the appropriate azimuthal symmetry.

3 Losses

Having established the shape of the modes in a confocal resonator, let us now discuss the losses that determine the Q -value and thereby the sensitivity of the resonator. There are three different loss mechanisms:

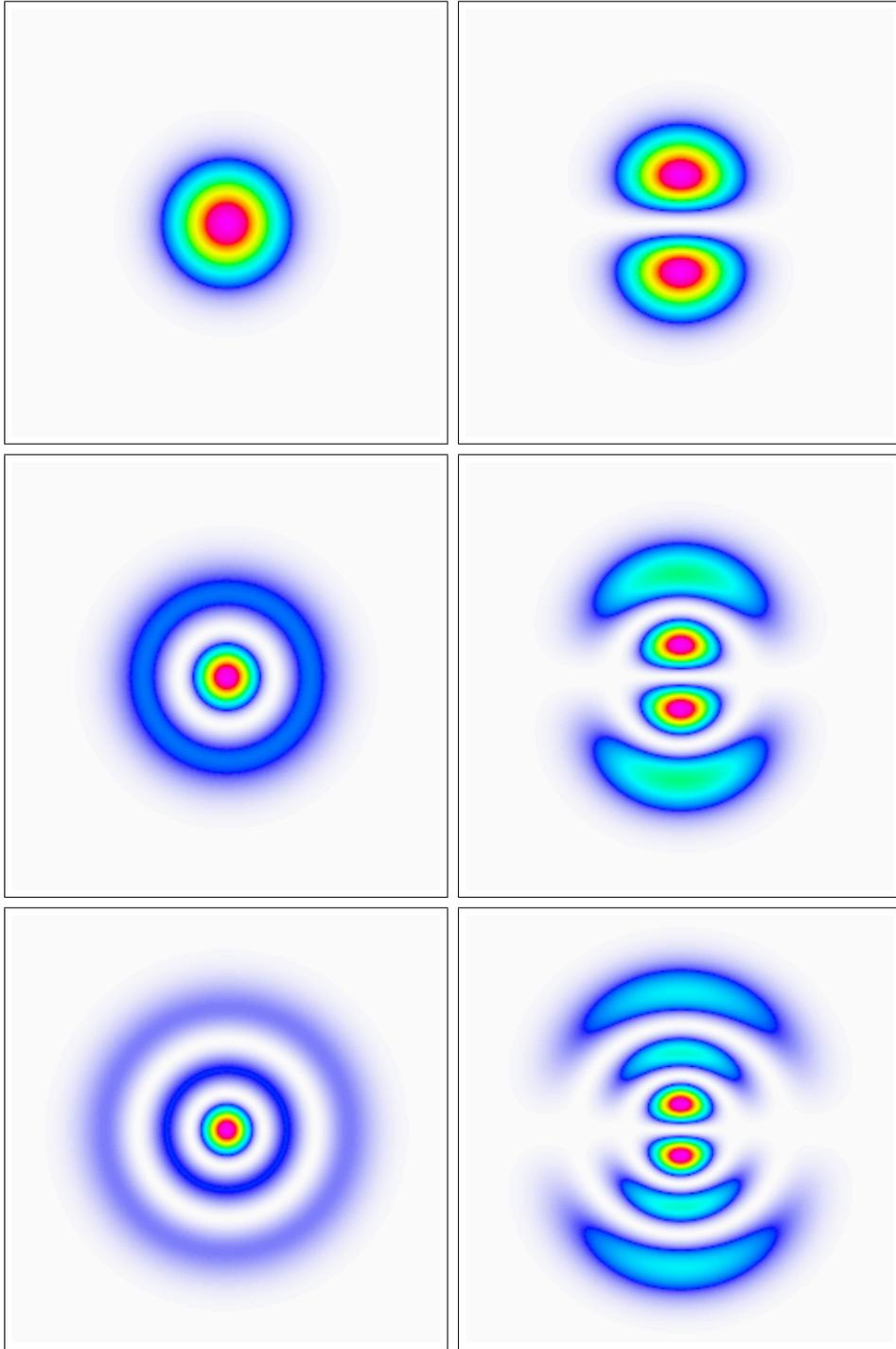


Figure 4: Intensity distribution in the $z=0$ plane for the following modes: $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, $(2,0)$, $(2,1)$. The distance covered is ± 40 mm in either direction.

- Diffraction losses,
- Losses due to output coupling,
- Losses due to finite conductivity.

We start by discussing the diffraction losses. Intuitively, they come from rays that miss the mirrors, if they have a finite extension. These losses are determined by the ratio of the gaussian beam waist $w(D/2)$ on the mirror and the mirror radius A . From eq. 1 we have $w(D/2) = \sqrt{D\lambda/\pi}$. Indeed, the relevant parameter for the diffraction losses is the Fresnel number N_F . For a confocal mirror, it is given by

$$N_F = \frac{A^2}{D\lambda} = \pi \left(\frac{A}{w(D/2)} \right)^2 . \quad (5)$$

The fractional power loss per mirror α_d for the $(0,0)$ mode is given in terms of the Fresnel number by

$$\alpha_d = 16\pi^2 N_F e^{-4\pi N_F} . \quad (6)$$

Fig. 3 shows how α_d varies as a function of N_F and it is clear that we need a Fresnel number around unity in order to have sufficiently small losses¹⁾. For $N_F = 1$ we derive a mirror radius of $A = \sqrt{D\lambda} = 39$ mm. For this value we get $\alpha_d = 5.5 \times 10^{-4}$. The corresponding Q_d -value is given by

$$Q_d = \frac{2\pi D}{\alpha_d \lambda} = 4.3 \times 10^4 . \quad (7)$$

Following [6] further, and still for the $(0,0)$ mode only, we calculate the losses due to a small central coupling hole used to extract power from the resonator for signal detection. We assume that the mirror has a thickness t and that the hole has a diameter d . The attenuation in the coupling hole is thus given by

$$\alpha_c t = \sqrt{\left(\frac{3.682t}{d}\right)^2 - \left(\frac{2\pi t}{\lambda}\right)^2} . \quad (8)$$

The extracted power is inversely proportional to

$$Q_c = \frac{27}{8\pi^2} \frac{\lambda^4 D^2}{d^6} e^{2r_0^2/w_0^2} e^{2\alpha_c t} \quad (9)$$

where r_0 is the offset of the coupling hole from the center. This implies that the extracted power is determined by the sixth power of the hole diameter. Larger holes are obviously advantageous to increase the power transmitted to the detector. However, the extracted power will be missing inside the cavity. The corresponding Q_h -value reduces to

$$Q_h = \frac{Q_c}{2e^{2\alpha_c t} - 1} . \quad (10)$$

There are thus two conflicting requirements. On the one hand, we want to have a large signal to detect but, on the other hand, we want to keep the Q_h -value as large as possible in order to maximize the sensitivity.

¹⁾ Note that eq. 6 strongly resembles the equation for the lifetime of a gaussian electron beam in the presence of limiting apertures, which depends in a similar way on the ratio of the beam size and the aperture limit.

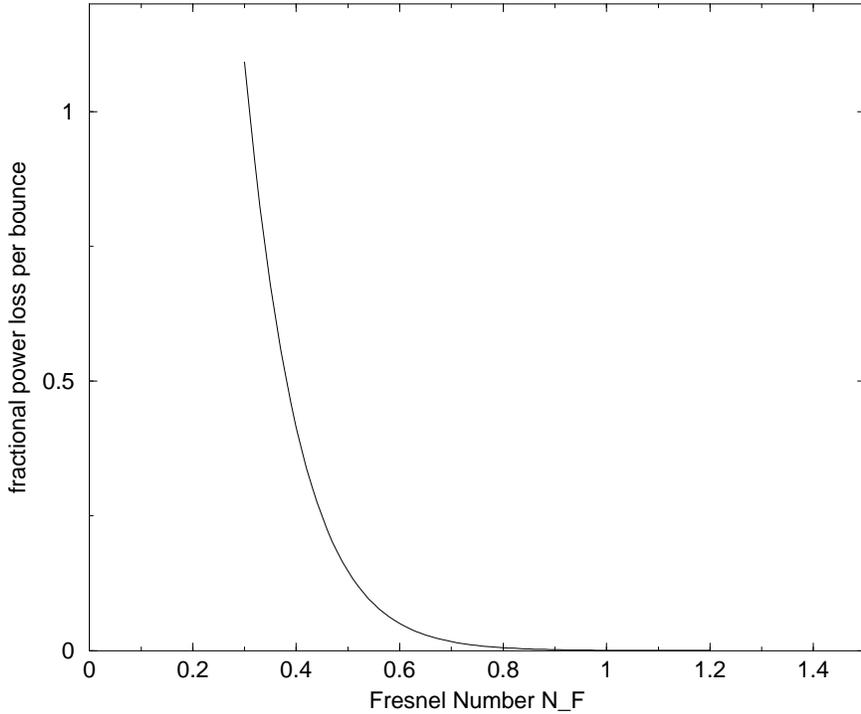


Figure 5: Fractional power loss α_d as a function of the Fresnel number N_F .

Inserting numbers from the previous example and assuming a central hole at $r_0 = 0$ with a diameter $d = 3$ mm and a depth $t = 2$ mm, we find an attenuation of $\alpha_c t = 2.4$, a Q_c -value of $Q_c = 1.2 \times 10^7$ and a total Q_h -value of $Q_h = 1.1 \times 10^5$.

Then, we have to consider the losses due to a finite conductivity. According to [6], they are determined by a geometry factor characterized by a resistance value

$$G = Z_0 \frac{\pi D}{2 \lambda} \quad (11)$$

where $Z_0 = 377 \Omega$ is the impedance of free space. For our configuration, $G = 2200 \Omega$. The other "ingredient" is the surface resistivity of the material $R_s = 1/\sigma\delta$, where σ is the conductance of the material and δ is the skin depth. For copper, $\sigma = 5.8 \times 10^7/\Omega\text{m}$ and, at 15 GHz, the skin depth is 3.8×10^{-7} m. This yields $R_s = 0.039 \Omega$. The corresponding Q_r -value is then given by

$$Q_r = \frac{G}{R_s} = 5.7 \times 10^4 . \quad (12)$$

Note that this result is the same for the fundamental and high-order modes. Since losses are described by the inverse of the Q -values, it is natural to combine the individual Q -values by adding their inverses, in order to obtain the total Q -value which is about 2×10^4 . Note that this result is the same for the fundamental and high-order modes.

The high Q -value will make the tolerances of the resonator quite tight, because we have $\Delta\lambda/\lambda \approx 1/Q$ and this corresponds to a change of the mirror distance D by about $2 \mu\text{m}$. Thermal variations could easily distort the resonator by this amount and may warrant a feedback system to keep the resonator dimensions constant. We need to address this more carefully later.²⁾

²⁾ Thanks to F. Caspers for pointing our attention to this.

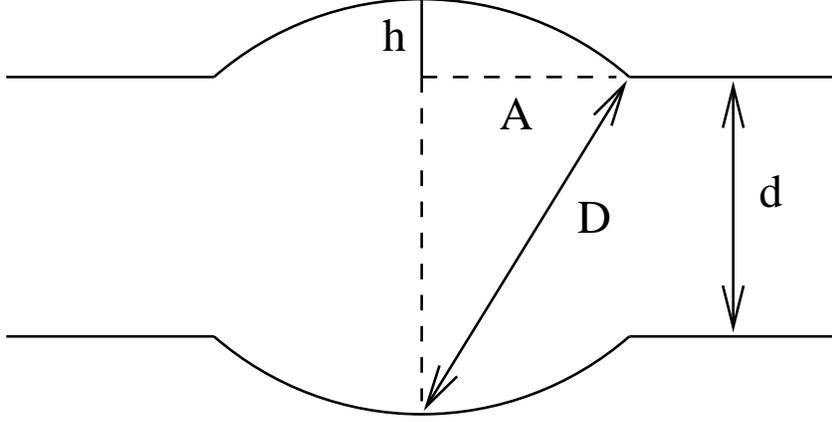


Figure 6: *Confocal resonator inserted into the beam pipe.*

4 Geometry

The confocal resonator has to be inserted into the beam pipe on either side. This determines the mirror radius A and also the elevation of the zenith of the mirror domes above the edges of the mirrors. It is given by $h = D - \sqrt{D^2 - A^2}$ as shown in Fig. 6. Thus, the height of the vacuum chamber d should be $d = D - 2h = -D + 2\sqrt{D^2 - A^2}$. The requirement $N_F = 1$ leads to $A^2 = D\lambda$ and the distance between the mirrors is given by the requirement that the electric field vanishes on the mirrors, i.e. $D = (l + 3/4)\lambda$.

Combining all these constraints, we get the following set of design equations

$$\begin{aligned} \frac{D}{\lambda} &= l + \frac{3}{4}, \\ \frac{d}{\lambda} &= \left(l + \frac{3}{4}\right) \left[2\sqrt{\frac{l - 1/4}{l + 3/4}} - 1 \right], \\ \frac{A}{\lambda} &= \sqrt{l + \frac{3}{4}}, \end{aligned} \quad (13)$$

where all relevant geometric quantities are expressed in terms of the wavelength λ and of the mode number l . In Table 1 we give the resulting parameters for 15 GHz ($\lambda = 20$ mm) and 30 GHz ($\lambda = 10$ mm).

For our initial design, we propose to build a 15 GHz resonator with $\lambda = 20$ mm and $l = 3$. The mechanical dimensions are shown in the upper section of Table 1, in the

Table 1: *Geometry parameters for the confocal resonator at 15 GHz and 30 GHz.*

	l	1	3	5	7	9
$\lambda = 20$ mm	D/mm	35.00	75.00	115.00	155.00	195.00
	d/mm	10.83	53.45	94.05	134.31	174.46
	A/mm	26.46	38.73	47.96	55.68	62.45
$\lambda = 10$ mm	D/mm	17.50	37.50	57.50	77.50	97.50
	d/mm	5.41	26.73	57.03	67.16	87.23
	A/mm	13.23	19.36	23.98	27.84	31.22

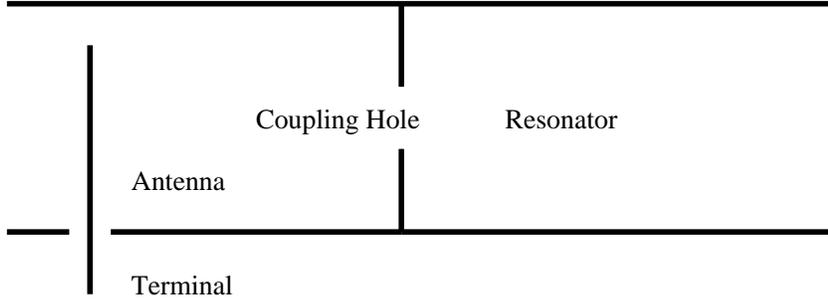


Figure 7: *The simple model used to analyze the coupling of the resonator to the antenna.*

column labelled 3. We choose a small l in order to maximize the transit time factor for the coupling to the beam. Another guiding principle was to keep the vacuum chamber as smooth as possible such that the resonator dome simply appears as a weak extrusion from the vacuum chamber. Finally the diameter of the dome should not dramatically exceed the height of the vacuum chamber to avoid excessive tapering of the vacuum chamber in order to accommodate for the confocal resonator.

5 Coupling to the beam

In this section, we calculate the transfer impedance from the beam current to the voltage detected at the terminal of the antenna. For this purpose, let us consider the corresponding kicker model and let us calculate the voltage seen by the beam for a given current oscillating in the antenna. Reciprocity then guarantees that the impedance derived for this kicker is equal to the transfer impedance [7, 8].

For definiteness sake, we consider a geometry in which the fields are excited by an antenna connected to a coaxial line. We consider a simplified model in which the antenna is situated in a rectangular waveguide with transverse dimensions a and b , that is coupled through a hole with radius r_0 to a resonator made up of the same waveguide and closed on the right-hand side. The corresponding layout is shown in Fig. 7. Note that the waveguide is open on the left-hand side. In this geometry, the radiated power P_a from the antenna towards one side of the waveguide is given by [9]

$$P_a = \frac{b}{2a} \frac{Z_0 I^2}{\sqrt{1 - (\lambda/2a)^2}} \quad (14)$$

where λ is the free space wavelength of the radiation and I is the current that drives the antenna. The microwaves emanating from the antenna will eventually meet the coupling hole of radius r_0 which, according to [9], can be modelled by a shunt inductance with an impedance

$$Z_{hole} = Z_w \frac{2\beta\alpha_m}{ab} = Z_w \frac{8\beta r_0^3}{3ab}, \quad (15)$$

where $\beta = \sqrt{k_0^2 - (\pi/a)^2}$ is the propagation constant of the TE₀₁ mode, Z_w is the impedance of the waveguide, and $\alpha_m = 4r_0^3/3$ is the magnetic polarization of a round coupling hole. The radius of the coupling hole r_0 can be chosen optimally by requiring that the impedance of the combined system (resonator + hole) is matched to the

impedance of the waveguide Z_w and is therefore given by [9]

$$r_0^3 = \frac{3}{16\pi} \frac{ab\lambda}{1 - (\lambda/2a)^2} \sqrt{\frac{\pi}{2Q}} . \quad (16)$$

In that case, all the energy radiated by the antenna is actually taken up by the resonator. Because of the finite Q -value, which accounts for the energy loss in the resonator, the energy does not pile up indefinitely inside the resonator.

In order to derive the order of magnitude for the involved quantities, we assume that the waveguide which is linked to the confocal resonator through the coupling hole has a quadratic cross section with a transverse length $a = b = 15$ mm, and thus above the cutoff for 20 mm microwaves. For the resonator, we use the Q -value of 20 000 found in section 3. For the optimum radius of the coupling hole, we obtain $r_0 = 1.6$ mm which is close to the value assumed in section 3 where the hole had a diameter of 3 mm.

The power coupled into the resonator will increase the total energy in the resonator U , which is approximately given by

$$U = \frac{\varepsilon_0}{2} \int_V E^2 dV \approx \frac{\pi\varepsilon_0}{4} D w_0^2 E_0^2 , \quad (17)$$

where E_0 is the electric field on the beam axis and w_0 is the waist radius defined in section 2. The amount of energy that escapes the resonator is given by the Q -value such that we can write an energy balance for the total energy U in the confocal resonator

$$\frac{dU}{dt} = -\frac{\omega}{Q}U + P_a . \quad (18)$$

At the equilibrium we have $dU/dt = 0$ and $U = QP_a/\omega$. Using eq. 17 and solving for the electric field on the beam axis E_0 we find

$$E_0^2 = \frac{2Q}{\pi} \left[\frac{Z_0 I}{(l + 3/4)\lambda} \right]^2 \frac{b/a}{\sqrt{1 - (\lambda/2a)^2}} \quad (19)$$

where we expressed D and w_0 in terms of λ and l . Taking the square root we get

$$E_0 = \sqrt{\frac{2Q}{\pi}} \frac{Z_0 I}{(l + 3/4)\lambda} \sqrt{\frac{b}{a}} \frac{1}{[1 - (\lambda/2a)^2]^{1/4}} , \quad (20)$$

which is the peak electric field on the beam axis. The energy change ΔE of an electron beam is given by the line integral over the electric field, which oscillates with the chosen frequency, here 15 GHz, while the particles cross the resonator waist

$$\Delta E = \int_{-\infty}^{\infty} E_0 e^{-z^2/w_0^2} \cos(2\pi z/\lambda) dz = \sqrt{\pi} w_0 E_0 e^{-(\pi w_0/\lambda)^2} . \quad (21)$$

The exponential reduction factor is quite dramatic ($e^{-5.9} = 2.8 \times 10^{-3}$ for the configuration discussed in this note). On the other hand, the waist w_0 is proportional to both λ and D such that the only way to improve the transit time factor is to make the resonator shorter (smaller D), but we are already at a short limit by using only 6 half periods ($l = 3$). Rewriting the reduction factor in eq. 21 in terms of l yields $\Delta E \propto e^{-\pi(l+3/4)/2}$ which clearly shows that a high mode number l is disadvantageous.

Finally, the transfer impedance Z_t can be deduced by inserting the peak electric field E_0 from eq. 20 into eq. 21

$$\Delta E = \sqrt{\frac{Q}{\pi(l + 3/4)\sqrt{1 - (\lambda/2a)^2}}} \sqrt{\frac{b}{a}} e^{-\pi(l+3/4)/2} Z_0 I = Z_t I . \quad (22)$$

Inserting values of $Q = 2 \times 10^4$, $a = b = 15$ mm, and $l = 3$ we obtain a transfer impedance $Z_t = 50 \Omega$.

By reciprocity, the voltage expected at the termination of the antenna when a beam goes through the resonator should be given by the same transfer impedance Z_t multiplied by the harmonic of the beam current at 15 GHz. In the CTF3 initial phase presently under commissioning at CERN, a bunch train consists of 4200 bunches spaced by 333 ps and it has a peak current of 3.5 A. The voltage at the antenna termination can thus reach about 150 V.

6 Coupling to waveguide modes

Here we discuss an intuitive and qualitative picture that describes the interaction between the waveguide modes propagating in the beam pipe and the resonator modes. When the incoming waveguide modes reach the entrance of the confocal resonator, the beam pipe ahead opens up and looks like free space. Therefore, the transition area acts like an aperture from which the mode diffracts into the open space ahead. The fraction of the mode that reaches the entrance to the beam pipe on the other side of the confocal resonator has escaped the resonator, but the part of the diffracted mode that misses the entrance to the beam pipe hits the mirrors and starts bouncing back and forth. It is thus coupled from the propagating mode to the resonator. This mechanism is similar to the one describing the diffraction losses of the resonator modes where we have $\alpha_d = 5.5 \times 10^{-4}$. Since all mechanical dimensions are approximately equal, the coupling efficiencies are approximately equal and we thus expect that a few times 10^{-4} of the power of the propagating mode is captured in the resonator. This makes it qualitatively clear that a high Q -value of the confocal resonator, which means small losses, implies that the coupling to the waveguide modes is weak.

7 Conclusion

In this note, we discuss the design of a confocal resonator pick-up using purely analytical means. We found that the coupling impedance to the beam current at the resonant frequency is tens of ohms and that waveguide modes are significantly attenuated. We found that a major loss mechanism which determines the Q -values is diffraction. This is not surprising, because the entire structure is small, when measured in units of the wavelength. In this regime, one might expect diffraction to play a big role. Other losses by output coupling or resistive losses are of comparable magnitude but somewhat smaller.

The confocal resonator pick-up could easily be modified to be used as a position monitor by adding coupling holes to both upper and lower domes and feeding the two signals to a hybrid to generate sum and difference signals. Moreover, even though the design in this note focussed on a single frequency, the resonator will accommodate other, higher frequencies as well. The excitation of the higher modes depends on the bunch

length and the pick-up could be used to monitor that. It is worthwhile to point out that the confocal resonator pick-up does not affect the trajectory of the beam, as a diagnostic device based on the detection of synchrotron radiation by a streak camera would. ³⁾

We need to point out that the paraxial approximation used in the derivation of eq. 1 is not entirely valid in the regime where the wavelength of the radiation is comparable to the geometric size of the structure and we therefore need to go beyond that approximation. An exact solution of the problem with two circular mirrors exists in the framework of the "Complex Source Point Theory". This will be investigated in the future in order to understand the influence of the approximations made in this note.

Another line of action is to simulate the confocal resonator numerically with codes that solve Maxwell's equations for a given set of boundary conditions, as well as simulations of the interaction with the beam. The insight found here will, however, be useful to check the sanity and the interpretation of the results.

Acknowledgements

We are grateful for comments and criticism from F. Caspers who served as a spiritus rector and also I. Syratchev, both at CERN.

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³⁾ We are grateful to F. Caspers for drawing our attention to these points.